SEGMENTS DIVIDED PROPORTIONALLY

POINTS ON LINE SEGMENT DEFINED BY RATIO

Given a line segment it is common to find the point halfway between the endpoints. It is the midpoint. It is also possible to find the point any fraction of the way between two endpoints.

Given a segment with end points (x_1, y_1) and (x_2, y_2). If t is the fraction of the distance between the end points, then the coordinates of the point (x, y) determined by t are given by:
$$x = x_1 + t(x_2 - x_1)$$
$$y = y_1 + t(y_2 - y_1)$$

Note that using $t = \frac{1}{2}$ gives an expression equivalent to the midpoint formula.

Example 1

Given the points A = (–2, 1) and B= (10, 10) find the coordinates of a point C that is $\frac{1}{3}$ of the distance from A to B. Using $t = \frac{1}{3}$ in the formula above:

$$x = x_1 + t(x_2 - x_1) = -2 + \frac{1}{3}(12) = -2 + 4 = 2$$
$$y = y_1 + t(y_2 - y_1) = 10 + \frac{1}{3}(9) = 10 + 3 = 13$$

$$C = (2, 13)$$

Example 2

Given the points A = (–2, 9) and B= (10, –5) find the coordinates of a point C such that $\frac{AC}{CB} = \frac{3}{2}$. If $\frac{AC}{CB} = \frac{3}{2}$ then C is located $\frac{3}{3+2} = \frac{3}{5}$ of the way between A and B so use $t = \frac{3}{5}$.

$$x = x_1 + t(x_2 - x_1) = -2 + \frac{3}{5}(12) = -2 + 7.2 = 5.2$$
$$y = y_1 + t(y_2 - y_1) = 9 + \frac{3}{5}(-14) = 9 + -8.4 = 0.6$$

$$C = (5.2, 0.6)$$

Problems

Find the coordinates of each point C described below.

1. $A = (3, 2)$, $B = (11, 10)$ and C is $\frac{1}{4}$ the distance from A to B.

2. $A = (-4, -9)$, $B = (11, -3)$ and C is $\frac{2}{3}$ the distance from A to B.

3. $A = (3, -1)$, $B = (-9, 8)$ and C is $\frac{2}{5}$ the distance from A to B.

4. $A = (-5, 8)$, $B = (6, 17)$ and C is $\frac{1}{4}$ the distance from B to A.

5. $A = (3, 2)$, $B = (15, 11)$ and $\frac{AC}{CB} = \frac{2}{1}$.

6. $A = (-5, 9)$, $B = (7, -3)$ and $\frac{AC}{CB} = \frac{1}{3}$.

7. $A = (-5, -9)$, $B = (-12, 8)$ and $\frac{AC}{CB} = 1$

8. $A = (0, 9)$, $B = (8, -2)$ and $\frac{AC}{CB} = 3$

9. $A = (-4, 3)$, $B = (8, -2)$ and B is the midpoint of $\overline{AC}$. Hint: $t > 1$.

10. $A = (4, 11)$, $B = (1, -3)$ and B is the midpoint of $\overline{AC}$. Hint: $t < 0$.

Answers

1. (5, 4)   2.   (6, –5)   3. (–1.8, –5.4)   4. $(3\frac{1}{4}, 14\frac{3}{4})$ 5.   (11, 8)

6.   (–2, 6)   7. $(-8\frac{1}{2}, -\frac{1}{2})$ 8. $(6, -\frac{1}{4})$ 9.   (20, –22)   10.   (–2, –17)