Investigation: Partitioning a Line Segment

The coordinate plane can be used to explore and design a variety of problems. In previous studies, you learned to represent, analyze and transform polygons in a coordinate plane. You studied their properties using ideas of distance and slope. In *An Orchard of Circles*, you developed the equation for all points an equal distance from a given center point. As you work on problems in this investigation, look for answers to this question:

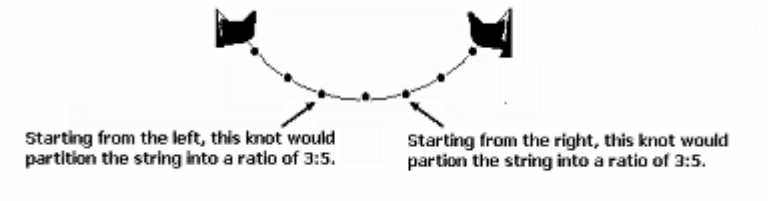
How do you find the point on a directed line segment for any given ratio?

Identifying the point that partitions a line segment into a given ratio requires an understanding of ratios, length and direction. By carefully studying simple cases such as a vertical line segment and a horizontal line segment, you can discover patterns that can be used for oblique line segments.

1. In this activity you will explore the idea of ratio using a piece of string 32 inches long.
   a. Begin by tying knots in the string to create 8 equal sections. Describe the strategy you used to create the 8 equal partitions and compare with others.
   b. Identify the knot that **partitions** the string into a ratio of 3:5. Explain your reasoning.
   c. What are the lengths of the two partitions? How do the lengths compare to the total length of 32 inches?
   d. Which knot partitions the string onto a ratio of 1:3? How do the lengths of the two partitions compare to the total length?
   e. Describe how to use the given ratio 1:3 to determine the lengths of each partition?

2. Modify the method you described in part 1e to determine the lengths of each partition for the following strings.
   a. Length of string is 27 inches and the knot partitions it into a ratio of 2:7
   b. Length of string is 48 feet and the knot partitions it into a ratio of 3:1
   c. Length of string is 56 yards and the knot partitions it into a ratio of 2:5
   d. Length of string is L and the knot partitions it into a ratio of a:b.

As you begin to explore the idea of partitioning a line segment in the coordinate plane it is important to recognize the difference between a line segment and a **directed line segment**. Consider the 32 inch string and the 8 partitions. If you hold the string in your hands, the knot that partitions the string onto a ratio of 3:5 depends on which end you use as the starting point.



A directed line segment means the line has a specific direction, meaning it starts at a specific endpoint and moves towards the other endpoint. Thus, the directed line segment AB will start at A and go to B while the directed line segment BA will start at B and go to A.

3. Explore how the use of ratio and lengths can be used to identify points on a coordinate plane that partition a directed line segment into a given ratio.
   a. Graph the points A(6, 2) and B(18, 2) and identify the point that partitions the directed line segment AB into a ratio of 1:3.
   b. What is the length of the directed line segment AB and how can the length be determined from the coordinates of the endpoints A and B?
   c. What is the length of the two partitions?
   d. Describe how to use the coordinate of the endpoint A to determine the location of the point that partitions the directed line segment into a ratio of 1:3. Compare your ideas with others.
   e. Graph the point C(18,10) and identify the point that partitions the directed line segment BC into a ratio of 1:3.
   f. What is the length of the directed line segment BC and how can that length be determined from the coordinates of the endpoints B and C?
   g. Describe how to use the coordinate of the endpoint B to determine the location of the point that partitions the directed line segment BC into a ratio of 1:3. Compare your ideas with others.

4. In activity 3 you explored a horizontal directed line segment AB and a vertical directed line segment BC.
   a. Using the results from problem 3, determine the point on directed line segment AC that partitions it into a ratio of 1:3.
   b. Provide evidence that the point does partition the directed line segment AC into a ratio of 1:3.

5. Use a graph and your understanding of ratio, length and direction to find the point that partitions each of the directed line segments into the given ratio.
   a. Directed line segment GH for G(3,4) and H(6,10) with a ratio of 1:2
   b. Directed line segment UM for U(-3,-2) and M(6,1) with a ratio of 2:1
   c. Directed line segment PK for P(5,-2) and K(-5,3) with a ratio of 1:3

6. Consider the circle $(x-2)^2+(y-5)^2=16$ and the circle $(x-8)^2+(y-13)^2=36$.
   a. What is the distance between the two centers?
   b. Find the point P on the boundary of the first circle and the boundary of the second circle that is also on the line segment connecting the two centers. Justify your reasoning.

Summarize the Mathematics

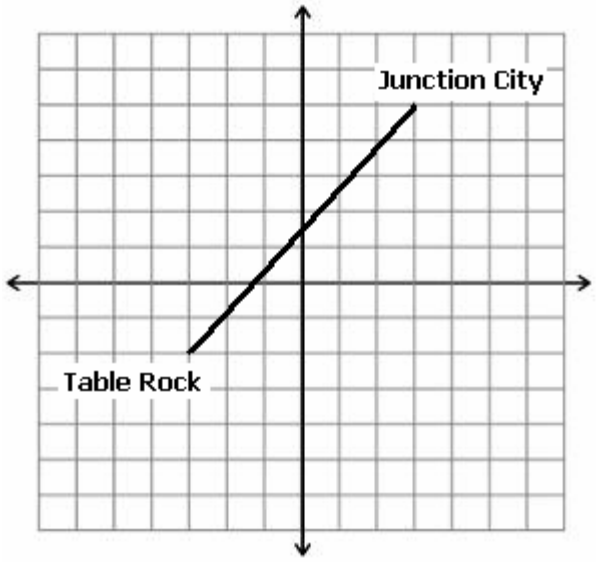
Consider point M that partitions the directed line segment AB for $A(x_1, y_1)$ and $B(x_2, y_2)$ into a ratio of g:h.

a) Describe and write expressions to
   i. determine the lengths of the vertical and horizontal components of AB
   ii. determine how far M is from the endpoint A both vertically and horizontally
   iii. determine the coordinates of point M

b) How would the answer to part iii change if the directed line segment was BA?

Check Your Understanding

a) The map shows a straight highway between two towns. Highway planners want to build two new rest stops between the towns so that the rest stops divide the highway into three equal parts. Find the coordinates of the points at which the rest stops should be built.



b) Find the coordinates of point F on the directed line segment DE for D(-2,5) and E(2,-3) that partitions it into a ratio of 4:1.