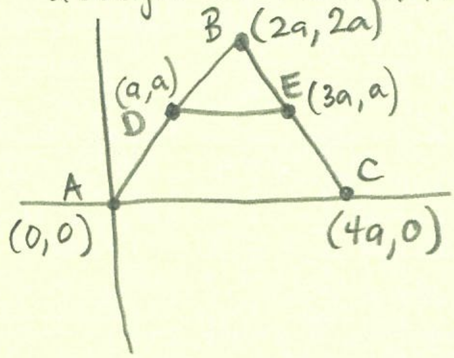
EX. Coordinate Proof of the Triangle Midsegment Conjecture

① Set up a triangle on the coordinate plane and assign coordinates.



② The midsegment will connect the midpoints of $\overline{AB}$ and $\overline{BC}$

$$\text{midpoint}(\overline{AB}) = \left(\frac{0+2a}{2}, \frac{0+2a}{2}\right) = (a, a)$$

$$\text{midpoint}(\overline{BC}) = \left(\frac{2a+4a}{2}, \frac{0+2a}{2}\right) = (3a, a)$$

③ Now prove that the length of the midsegment is half the length of $\overline{AC}$

length $\overline{DE} = 2a$ length of $\overline{AC} = 4a$

$$\frac{2a}{4a} = \frac{1}{2}$$

④ Now show that midsegment $\overline{DE}$ is parallel to base $\overline{AC}$. (The slopes must be equal)

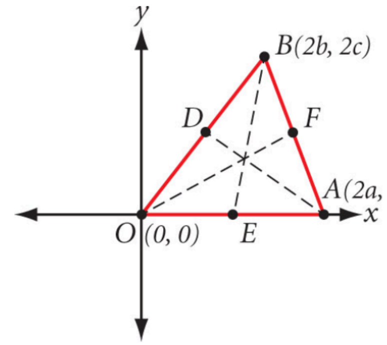
$$m_{\overline{DE}} = \frac{a-a}{3a-a} = \frac{0}{2a} = 0 \qquad m_{\overline{AC}} = \frac{0-0}{4a-0} = \frac{0}{4a} = 0 \quad \checkmark$$

Therefore, the Triangle Midsegment Conjecture is true.

"A midsegment of a triangle is parallel to the third side and half the length of the third side.

Honors Geometry Assignment – Proofs with Medians and Midsegments

7. DEVELOPING PROOF You prove that the medians of a triangle are concurrent the same way you found the centroid in the example except you use letters instead of numbers. To make the manipulating of symbols easier, place the points on the coordinate grid at "nice" locations (usually on one of the axes). Locate the points of your diagram so that they correspond to the given information, and you are not assuming extra properties. The plan for the coordinate proof is started for you. Complete the proof to show that the medians of a triangle are concurrent.



Given: $\triangle OAB$, without loss of generalitly, vertices $O(0, 0)$, $A(2a, 0)$, $B(2b, 2c)$

Show: The three medians of a triangle are concurrent.

Plan:

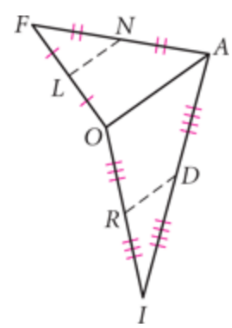
- Find the midpoint of $\overline{OA}$, $\overline{OB}$, and $\overline{AB}$ $\{D, E, F\}$
- Find the equations of medians $\overline{OF}$, $\overline{AD}$, and $\overline{BE}$.
- Find the point of intersection of medians $\overline{OF}$ and $\overline{AD}$.
- Find the point of intersection of of medians $\overline{OF}$ and $\overline{BE}$.
- If the point of intersection of medians $\overline{OF}$ and $\overline{AD}$ and the point of intersection of medians $\overline{OF}$ and $\overline{BE}$ are the same, then you have proved the three medians are concurrent!

8. DEVELOPING PROOF Copy and complete the flowchart to show that $\overline{LN} \parallel \overline{RD}$.

Given: Midsegment $\overline{LN}$ in $\triangle FOA$

Midsegment $\overline{RD}$ in $\triangle IOA$

Show: $\overline{LN} \parallel \overline{RD}$



Flowchart Proof

1	$\triangle FOA$ with midsegment $\overline{LN}$	Given
2	$\triangle IOA$ with midsegment $\overline{RD}$	Given
3	$\overline{LN} \parallel \overline{OA}$	?
4	?	Triangle Midsegment Conjecture
5	?	Two lines parallel to the same line are parallel