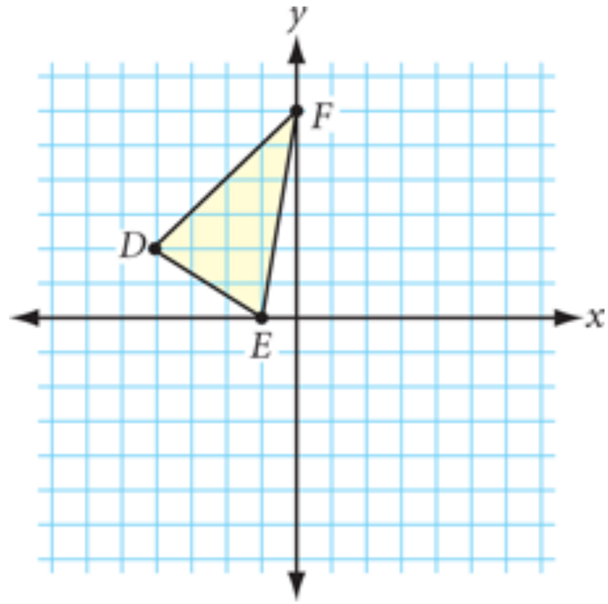
Here is another example of a composition of rotation and reflection transformations.

EXAMPLE C

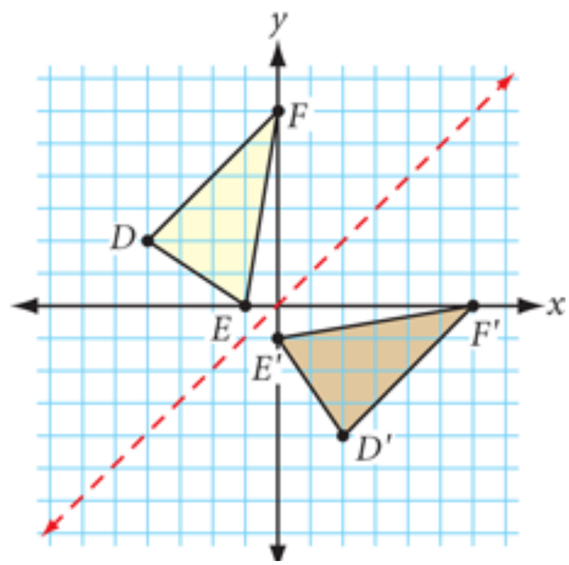
Given ΔDEF with vertices: $D(-4, 2)$, $E(-1, 0)$, $F(0, 6)$. Reflect ΔDEF across the line $y = x$ to create $\Delta D'E'F'$. Rotate ΔDEF 90° clockwise about the origin to create $\Delta D''E''F''$. Reflect $D''E''F''$ across the x-axis to create $\Delta D'''E'''F'''$.



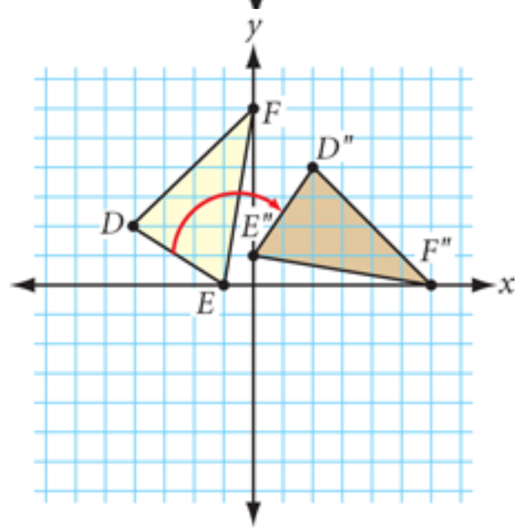
a) What is the transformation rule, $(x, y) \rightarrow (?, ?)$, that transforms ΔDEF to $\Delta D'E'F'$?
b) What are the coordinates of the vertices of $\Delta D'E'F'$?
c) What are the coordinates of the vertices of $\Delta D''E''F''$?
d) What is the transformation rule, $(x, y) \rightarrow (?, ?)$, that transforms ΔDEF to $\Delta D''E''F''$?
e) What are the coordinates of the vertices of $\Delta D'''E'''F'''$?
f) What is the single transformation rule that takes $\Delta D''E''F''$ onto $\Delta D'''E'''F''$?
g) What is the single transformation rule that takes ΔDEF onto $\Delta D'''E'''F''$?

Solution

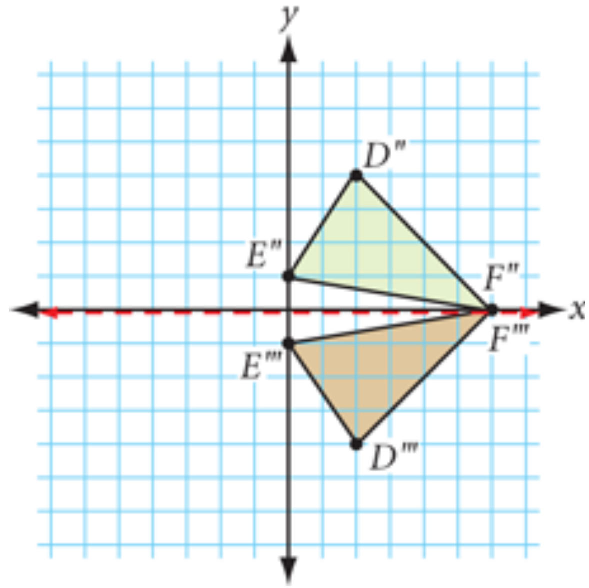
Step 1 Draw ΔDEF on a set of axes and relocate its vertices by reflecting ΔDEF across the line $y = x$ to get $\Delta D'E'F'$. Earlier you discovered that a reflection across the line $y = x$ switches the x- and y-values of each ordered pair thus it is equivalent to the ordered pair rule $(x, y) \rightarrow (y, x)$. Therefore $D(-4, 2)$, $E(-1, 0)$, $F(0, 6)$ become $D'(2, -4)$, $E'(0, -1)$, $F'(6, 0)$.



Step 2 Earlier you discovered that a rotation of 90° clockwise is equivalent to the ordered pair rule $(x, y) \rightarrow (y, -x)$. Relocating the vertices of ΔDEF using the ordered pair rule gives us vertices $D''(2, 4)$, $E''(0, 1)$ and $F''(6, 0)$.



Step 3 Earlier you discovered that a reflection across the *x*- axis is equivalent to the ordered pair rule $(x, y) \rightarrow (x, -y)$. Relocating the vertices of $\Delta D''E''F''$ using the ordered pair rule gives us vertices $D'''(2, 4)$, $E'''(0, 1)$, and $F'''(6, 0)$.



Step 4 The ordered pair rule that takes ΔDEF onto $\Delta D''E''F''$ is $(x, y) \rightarrow (y, -x)$. The ordered pair rule that takes $\Delta D''E''F''$ onto $\Delta D'''E'''F'''$' is $(x, y) \rightarrow (x, -y)$. Thus the ordered pair rule that takes ΔDEF onto $\Delta D'''E'''F'''$ is $(x, y) \rightarrow (y, -x) \rightarrow (y, x)$, or simply $(x, y) \rightarrow (y, x)$. Notice, this is the same as the ordered pair rule $(x, y) \rightarrow (y, x)$ that reflected ΔDEF onto $\Delta D'E'F'$.
$(x, y) \rightarrow (x, -y)$

Conclusion:
The composition of a 90° CW(clockwise) rotation and a reflection across the x-axis is the same as a single reflection across the line y=x.